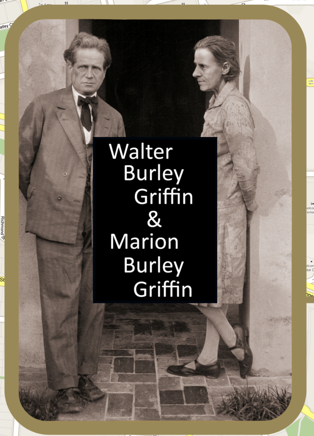

To be Designed or to be Generated – That is the Question!

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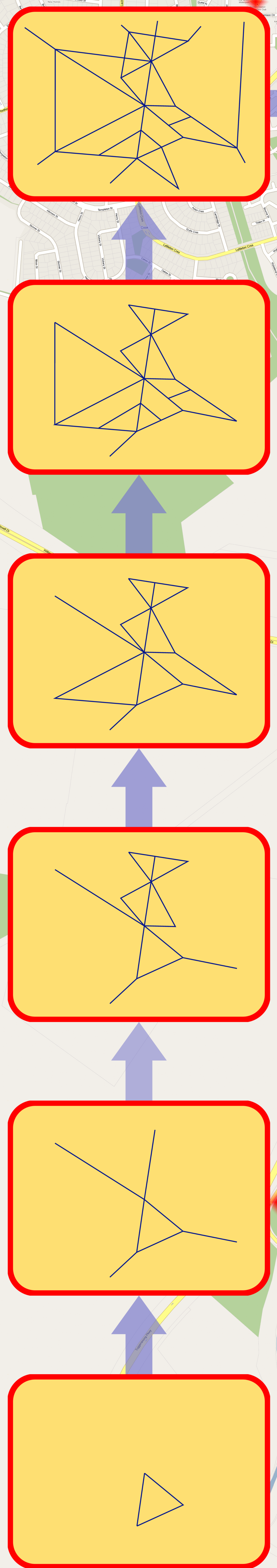

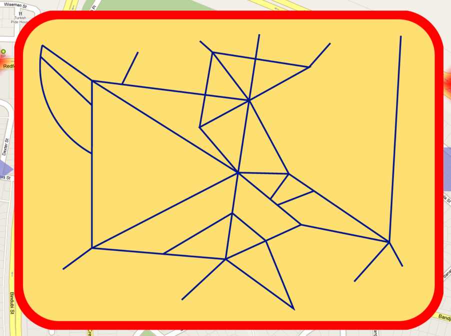

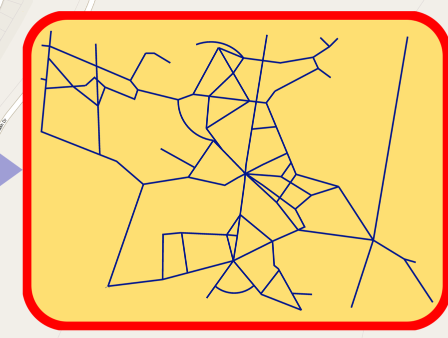

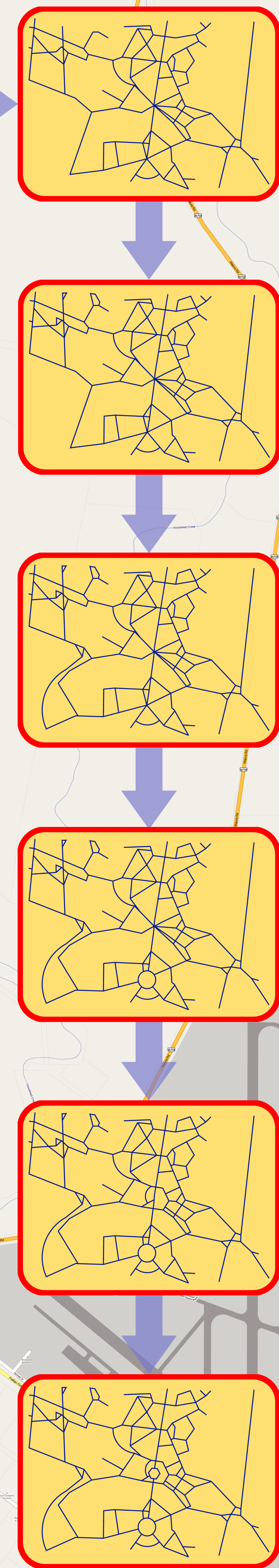

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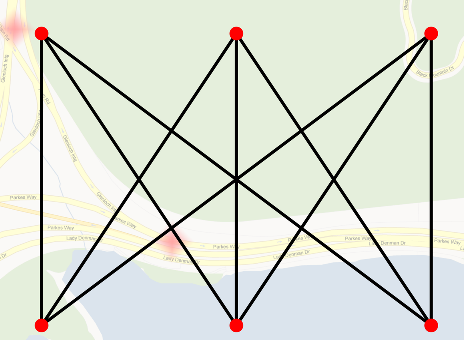
Introduction

Since last century, finding the structure of different families of graphs has been of great importance, not only in combinatorics but also for other fields. Graphs have been used for many theoretical and practical purposes such as Nanotechnology, Network Design, Chemistry and Numerical Analysis.

A **graph** is a model of a set of objects and the relations between them, which are visualised by points and arcs (or lines) between them.



A graph with 6 vertices and 9 edges

Planar graphs and regular graphs are the most studied families in term of graph generation because of their wide application. A **planar graph** is a graph which can be drawn without any of the lines crossing. Additionally, a graph is k**-regular** if each object has a relationship with k objects. Figure 1 shows two planar regular graphs in Red and Blue.

The **dual** of a planar graph G is a graph which has a point for each face of G. In addition, for each pair of faces which are adjacent, their corresponding points are joined with an arc. The graphs Red and Blue in Figure 1 are dual to each other (as represented by the dotted arcs). Planar graphs whose faces are all of size **3**, **4** and **5** are called **triangulations**, **quadrangulations** and **pentangulations**.

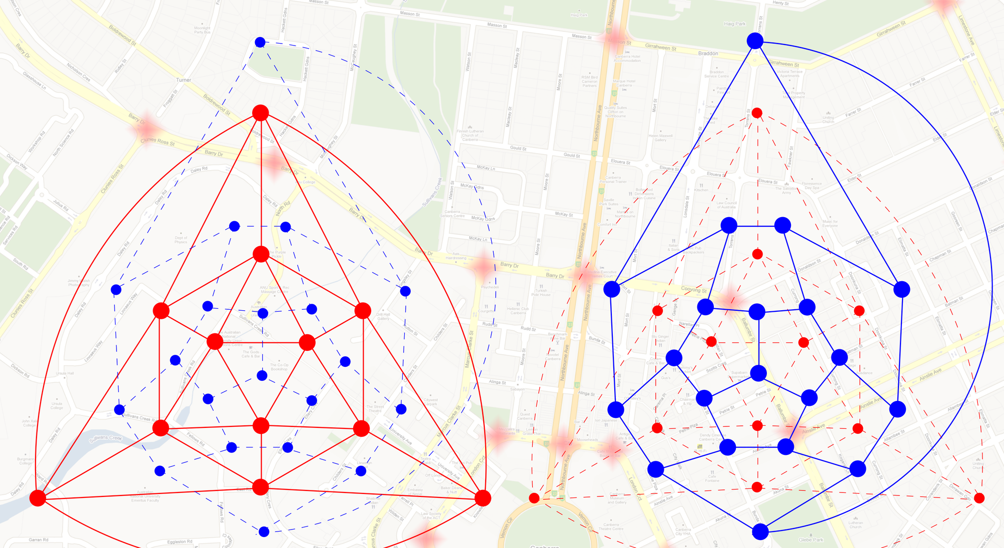


Fig. 1: A 5-regular triangulation & its dual 3-regular pentangulation

The degree (that is the number of relationships) of each point in a planar regular graph can be at most 5. Thus far, methods have been developed for generating 3-regular, 4-regular and 5-regular graphs as well as triangulations and quadrangulations.

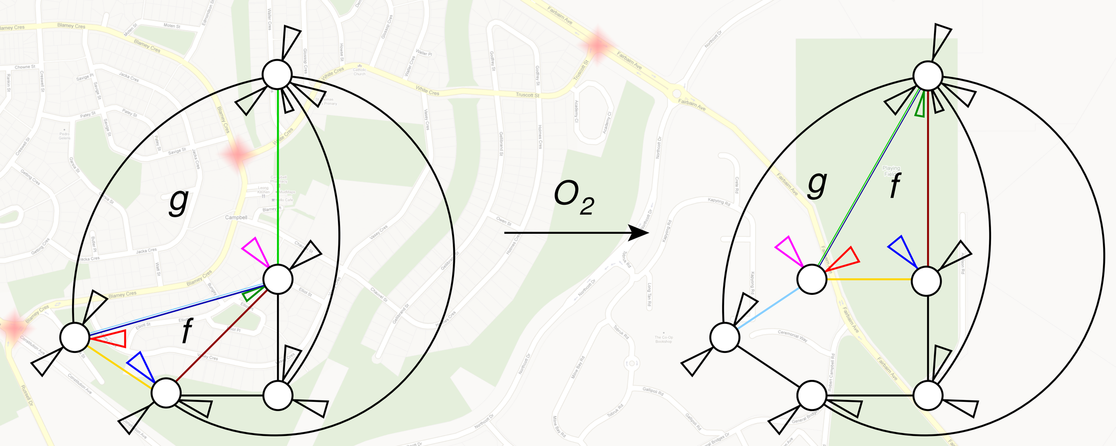
What does "Generation" Mean?

So far, one of the most successful graph generation methods has been **recursive generation**. This method uses an initial set of graphs and a collection of operations on graphs, each of which creates a larger graph from a smaller one. As such, this allows us to generate a family of graphs, where each graph in this family is the result of applying an operation to an initial or a previously generated graph.

Generation of Pentangulations

Our aim is to produce a generating method for the set of pentangulations with a specified number (n) of faces. We approach our goal in two steps:

1. We generate a triangulation with n faces using Bowen's method[1].
2. Repetitively we convert each triangle to a pentagon using one of the operations demonstrated in the figure below.



Theorem: Any pentangulation with n faces can be constructed from at least one triangulation with the same number of faces by n applications of operations O_1 and O_2.

[1] R. Bowen and S. Fisk, Generation of triangulations of the sphere, Math. Comput., **21** (1967) 250–252.